FYJC - MATHEMATICS & STATISTICS

HIGHLIGHTS

- ✓ *Solution to all questions*
- ✓ *solutions are put in way the student is expected to reproduce in the exam*
- ✓ *taught in the class room the same way as the solution are put up here . That makes the student to easily go through the solution & prepare him/herself when he/she sits back to revise and recall the topic at any given point of time .*
- ✓ *lastly, if student due to some unavoidable reasons , has missed the lecture , will not have to run here and there to update his/her notes .*
- ✓ *however class room lectures are must for easy passage of understanding & learning the minuest details of the given topic*

PAPER - I

CIRCLE

Compiled & Conducted @ JKSC

CONICS – CIRCLE

- ✓ Standard form of the circle : $x^2 + y^2 = r^2$

- ✓ Given centre C ≡ (h,k) & radius = r , equation of the circle can be generated using

  Center Radius form

  $(x - h)^2 + (y - k)^2 = r^2$

- ✓ Given $A(x_1,y_2)$, $B(x_2,y_2)$ are the ends of the diameter , equation of the circle can be generated using

  Diameter Form

  $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

- ✓ In general , equation of the circle

  $x^2 + y^2 + 2gx + 2fy + c = 0$, where

  $C \equiv (-g,-f)$ $R = \sqrt{g^2 + f^2 - c}$

Q1.

01. find the equation of the circle with center (2 , –3) and passing through (–1 , 2)

    ans : $x^2 + y^2 - 4x + 6y - 21 = 0$

02. find the equation of the circle with center (1 , –2) and passing through (5 , 3)

    ans : $x^2 + y^2 - 2x + 4y - 36 = 0$

03. find the equation of the circle with center (–2 , 3) and passing through (1 , 7)

    ans : $x^2 + y^2 + 4x - 6y - 12 = 0$

04. find the equation of the circle with center (1⁄2 , 3⁄2) and radius 3

    ans : $2x^2 + 2y^2 - 2x - 6y - 13 = 0$

Q2.

01. find equation of circle with radius 5 and concentric with circle $x^2 + y^2 + 4x - 6y = 0$

    ans : $x^2 + y^2 + 4x - 6y - 12 = 0$

02. find equation of circle with radius 5 and concentric with circle $x^2 + y^2 - 6x - 4y - 3 = 0$

    ans : $x^2 + y^2 - 6x - 4y - 12 = 0$

03. find equ. of circle concentric with $x^2 + y^2 - 2x - 6y - 7 = 0$ and area 616 sq. units

    ans : $x^2 + y^2 - 2x - 6y - 186 = 0$

04. find equ. of circle concentric with $x^2 + y^2 - 6x + 60 = 0$ and circumference 4π

    ans : $x^2 + y^2 - 6x + 5 = 0$ **(MAR 2016)**

05. Find centre and radius of the circle : $2x^2 + 2y^2 - 2x - 8y - 13 = 0$ **(MAR 2014)**

ans : $C(^1/_2 , 2)$, $r = \sqrt{43} / 2$

06. Find centre and radius of the circle : $3x^2 + 3y^2 - 6x + 4y - 42 = 0$

ans : $C(1 , {}^{-2}/_3)$, $r = \sqrt{22} / 3$

07. find the center and the radius of the circle : $(x - 3)(x - 5) + (y - 1)(y - 7) = 0$

ans : $C(4 , 4)$, $r = \sqrt{10}$

Q3.

01. find equation of the circle having centre (7,–2) and touching the x – axis

ans : $x^2 + y^2 - 14x + 4y + 49 = 0$

02. find equation of the circle having centre (–5,2) and touching the y – axis

ans : $x^2 + y^2 + 10x - 4y + 4 = 0$

03. find equation of the circle having radius = 1 and touching the x – axis at (–4,0)

ans : $x^2 + y^2 + 8x \pm 2y + 16 = 0$

Q4.

01. Find circle touching both the axes and having radius 7

ans : $x^2 + y^2 \pm 14x \pm 14y + 49 = 0$

02. find equation of the circle touching both axes and passing through (1,2)

ans : $x^2 + y^2 - 2x - 2y + 1 = 0$, $x^2 + y^2 - 10x - 10y + 25 = 0$

03. find equation of the circle touching both axes and passing through (–9,8)

ans : $x^2 + y^2 + 10x - 10y + 25 = 0$, $x^2 + y^2 + 58x - 58y + 841 = 0$

Q5.

01. Find equation of circle with center (4,3) & touching $5x - 12y - 10 = 0$

ans : $x^2 + y^2 - 8x - 6y + 21 = 0$

02. Find equation of circle with center (3,1) & touching $8x - 15y + 25 = 0$

ans : $x^2 + y^2 - 6x - 2y + 6 = 0$

Q6.

01. Find equation of circle passing through (4 , 6) ; (–3 , 5) & (5 , –1)

ans : $x^2 + y^2 – 2x – 4y – 20 = 0$

02. Find equation of circle passing through (4 , 1) ; (–3 , –6) & (–2 , 1) **(MAR 2013)**

ans : $x^2 + y^2 – 2x + 6y – 15 = 0$

03. Find equation of circle passing through (4 , 1) ; (6 , 5) & whose center lies on $4x + y = 16$

ans : $x^2 + y^2 – 6x – 8y + 15 = 0$

04. Find equation of circle passing through (1 , –4) ; (5 , 2) & whose center lies on $x – 2y + 9 = 0$

ans : $x^2 + y^2 + 6x – 6y – 47 = 0$

Q7.

01. find equation of circle passing through (1 , 9) & touching $3x + 4y + 6 = 0$ at (–2 , 0)

ans : $x^2 + y^2 – 2x – 8y – 8 = 0$

02. find equation of circle passing through (–1 , –3) & touching $4x + 3y – 12 = 0$ at (3 , 0)

ans : $x^2 + y^2 – 2x + 3y – 3 = 0$

Q8.

01. Find equation of circle with center (3 , –1) and which cuts off a chord of length 6 on line $2x – 5y + 18 = 0$

ans : $x^2 + y^2 – 6x + 2y – 28 = 0$

02. Find equation of circle with center (1 , 4) and which cuts off a chord of length 6 on line $3x + 4y + 1 = 0$ **(MAR 2014)**

ans : $x^2 + y^2 – 2x – 8y – 8 = 0$

03. Find the length of intercept made by circle $x^2 + y^2 – 2x – 8y – 8 = 0$ on the line $3x + 4y + 1 = 0$

ans : 6

04. Find the length of intercept made by circle $x^2 + y^2 – 6x + 4y – 12 = 0$ on the line $4x – 3y + 2 = 0$

ans : 6

MARCH – 2015

line $2x - y + 6 = 0$ meets the circle $x^2 + y^2 - 2x - 12 = 0$ at A and B . Find the equation of circle on AB as diameter

MARCH – 2017

Find equation of circle passing through point of intersection of the lines $x + 3y = 0$ and $2x - 7y = 0$ and whose centre is the point of intersection of the lines $x + y + 1 = 0$ and $x - 2y + 4 = 0$

SOLUTION SET

Q1.

01. find the equation of the circle with center (2 , –3) and passing through (–1 , 2)

SOLUTION

STEP 1 :

$$r = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(2 + 1)^2 + (-3 - 2)^2}$$

$$= \sqrt{9 + 25}$$

$$= \sqrt{34}$$

STEP 2 : $C(2,-3)$, $r = \sqrt{34}$

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 2)^2 + (y + 3)^2 = (\sqrt{34})^2$$

$$x^2 - 4x + 4 + y^2 + 6y + 9 = 34$$

$$x^2 + y^2 - 4x + 6y + 13 - 34 = 0$$

$$x^2 + y^2 - 4x + 6y - 21 = 0$$ ..... equation of the circle

02. find the equation of the circle with center (1 , –2) and passing through (5 , 3)

SOLUTION

STEP 1 :

$$r = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(1 - 5)^2 + (-2 - 3)^2}$$

$$= \sqrt{16 + 25}$$

$$= \sqrt{41}$$

STEP 2 : $C(1,-2)$, $r = \sqrt{41}$

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 1)^2 + (y + 2)^2 = (\sqrt{41})^2$$

$$x^2 - 2x + 1 + y^2 + 4y + 4 = 41$$

$$x^2 + y^2 - 2x + 4y + 5 - 41 = 0$$

$$x^2 + y^2 - 2x + 4y - 36 = 0$$ ..... equation of the circle

03. find the equation of the circle with center (–2 , 3) and passing through (1 , 7)

SOLUTION

STEP 1 :

$$r = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(-2 - 1)^2 + (3 - 7)^2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25} = 5$$

STEP 2 : $C(-2,3)$, $r = 5$

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x + 2)^2 + (y - 3)^2 = (5)^2$$

$$x^2 + 4x + 4 + y^2 - 6y + 9 = 25$$

$$x^2 + y^2 + 4x - 6y + 13 - 25 = 0$$

$$x^2 + y^2 + 4x - 6y - 12 = 0$$ ..... equation of the circle

04. find the equation of the circle with center (1/2 , 3/2) and radius 3

SOLUTION

STEP 1 : $C(1/2 , 3/2)$, $r = 3$

$$(x - h)^2 + (y - k)^2 = r^2$$

$$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 = 3^2$$

$$\left(\frac{2x - 1}{2}\right)^2 + \left(\frac{2y - 3}{2}\right)^2 = 9$$

$$\frac{4x^2 - 4x + 1}{4} + \frac{4y^2 - 12y + 9}{4} = 9$$

$$4x^2 + 4y^2 - 4x - 12y + 10 = 36$$

$$4x^2 + 4y^2 - 4x - 12y - 26 = 0$$

$$2x^2 + 2y^2 - 2x - 6y - 13 = 0$$ ..... equation of the circle

Q2.

01. find equation of circle with radius 5 and concentric with circle $x^2 + y^2 + 4x - 6y = 0$

SOLUTION

STEP 1 : $x^2 + y^2 + 4x - 6y = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = 4 \; ; \; 2f = -6$

$g = 2 \; ; \; f = -3 \; ; \; c = 0$

$C \equiv (-g , -f)$

$\equiv (-2 , 3)$

STEP 2 : $C(-2,3) \; , \; r = 5$

$(x - h)^2 + (y - k)^2 = r^2$

$(x + 2)^2 + (y - 3)^2 = (5)^2$

$x^2 + 4x + 4 + y^2 - 6y + 9 = 25$

$x^2 + y^2 + 4x - 6y + 13 - 25 = 0$

$x^2 + y^2 + 4x - 6y - 12 = 0$ ..... equation of the circle

02. find equation of circle with radius 5 and concentric with circle $x^2 + y^2 - 6x - 4y - 3 = 0$

SOLUTION

STEP 1 : $x^2 + y^2 - 6x - 4y - 3 = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -6 \; ; \; 2f = -4$

$g = -3 \; ; \; f = -2 \; ; \; c = 0$

$C \equiv (-g , -f)$

$\equiv (3,2)$

STEP 2 : $C(3,2) \; , \; r = 5$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 3)^2 + (y - 2)^2 = (5)^2$

$x^2 - 6x + 9 + y^2 - 4y + 4 = 25$

$x^2 + y^2 - 6x - 4y + 13 - 25 = 0$

$x^2 + y^2 - 6x - 4y - 12 = 0$ ..... equation of the circle

03. find equ. of circle concentric with $x^2 + y^2 - 2x - 6y - 7 = 0$ and area 616 sq. units

SOLUTION

STEP 1 : $x^2 + y^2 - 2x - 6y - 7 = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -2$; $2f = -6$

$g = -1$; $f = -3$; $c = 0$

$C \equiv (-g, -f) \equiv (1,3)$

STEP 2 : area = 616

$\pi r^2 = 616$

$r^2 = \frac{616}{\pi}$

$r^2 = \frac{616 \times 7}{22} = 196$

$r = 14$

STEP 3 : $C(1,3)$, $r = 14$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 1)^2 + (y - 3)^2 = (14)^2$

$x^2 - 2x + 1 + y^2 - 6y + 9 = 196$

$x^2 + y^2 - 2x - 6y + 10 - 196 = 0$

$x^2 + y^2 - 2x - 6y - 186 = 0$ ..... equation of the circle

04. find equ. of circle concentric with $x^2 + y^2 - 6x + 60 = 0$ and circumference is 4π

SOLUTION

STEP 1

$x^2 + y^2 - 6x + 60 = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -6$; $2f = 0$; $c = {}^{-11}/_{3}$

$g = -3$; $f = 0$; $c = {}^{-11}/_{3}$

$C \equiv (-g, -f) \equiv (3, 0)$

STEP 2

circumference = 4π

$2\pi r = 4\pi$

$r = 2$

STEP 3 : $C(3,0)$, $r = 2$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 3)^2 + (y - 0)^2 = 2^2$

$x^2 - 6x + 9 + y^2 = 4$

$x^2 + y^2 - 6x + 5 = 0$

05. Find centre and radius of the circle : $2x^2 + 2y^2 - 2x - 8y - 13 = 0$

SOLUTION

$$2x^2 + 2y^2 - 2x - 8y - 13 = 0$$

$$x^2 + y^2 - x - 4y - \frac{13}{2} = 0$$

On comparing with

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$2g = -1 \; ; \quad 2f = -4 \; ; \quad c = -13/2$$

$$g = -\frac{1}{2} \; ; \quad f = -2 \; ; \quad c = -13/2$$

$$C \equiv (-g, -f) \qquad R = \sqrt{g^2 + f^2 - c}$$

$$\equiv \left(\frac{1}{2}, 2\right) \qquad = \sqrt{\frac{1}{4} + 4 + \frac{13}{2}}$$

$$= \sqrt{\frac{1 + 16 + 26}{4}} \qquad = \frac{\sqrt{43}}{2}$$

06. Find centre and radius of the circle : $3x^2 + 3y^2 - 6x + 4y - 3 = 0$

SOLUTION

$$3x^2 + 3y^2 - 6x + 4y - 3 = 0$$

$$x^2 + y^2 - 2x + \frac{4y}{3} - 1 = 0$$

On comparing with

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$2g = -2 \; ; \quad 2f = \frac{4}{3} \; ; \quad c = -1$$

$$g = -1 \; ; \quad f = -\frac{2}{3} \; ; \quad c = -1$$

$$C \equiv (-g, -f) \qquad R = \sqrt{g^2 + f^2 - c}$$

$$\equiv \left(1, -\frac{2}{3}\right) \qquad = \sqrt{1 + \frac{4}{9} + 1}$$

$$= \sqrt{\frac{9 + 4 + 9}{4}}$$

$$= \frac{\sqrt{22}}{3}$$

07. find the center and the radius of the circle : $(x - 3)(x - 5) + (y - 1)(y - 7) = 0$

$(x - 3)(x - 5) + (y - 1)(y - 7) = 0$

$x^2 - 8x + 15 + y^2 - 8y + 7 = 0$

$x^2 + y^2 - 8x - 8y + 22 = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -8 \; ; \; 2f = -8 \; ; \; c = 22$

$g = -4 \; ; \; f = -4 \; ; \; c = 22$

$C \equiv (-g , -f)$ $\quad R = \sqrt{g^2 + f^2 - c}$

$\equiv (4,4)$ $\quad = \sqrt{16 + 16 - 22}$

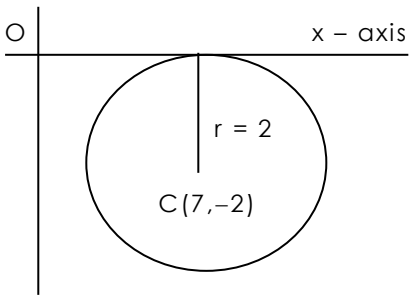
$= \sqrt{10}$

Q3.

01. find equation of the circle having centre (7,–2) and touching the x – axis

SOLUTION

$r = 2$ ..... (REFER DIAGRAM)



$C(7,-2)$, $r = 2$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 7)^2 + (y + 2)^2 = (2)^2$

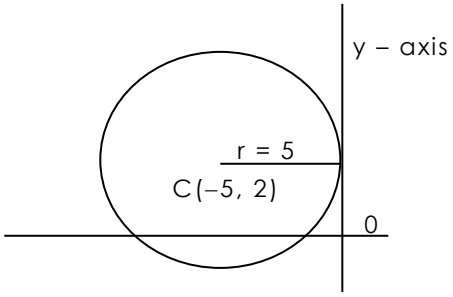
$x^2 - 14x + 49 + y^2 + 4y + 4 = 4$

$x^2 + y^2 - 14x + 4y + 49 = 0$ ..... equation of the circle

02. find equation of the circle having centre (–5,2) and touching the y – axis

SOLUTION

$r = 5$ ..... (REFER DIAGRAM)



$C(-5, 2)$, $r = 5$

$(x - h)^2 + (y - k)^2 = r^2$

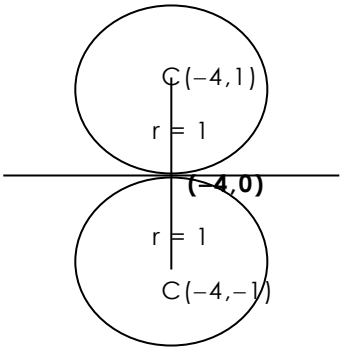
$(x + 5)^2 + (y - 2)^2 = (5)^2$

$x^2 + 10x + 25 + y^2 - 4y + 4 = 25$

$x^2 + y^2 + 10x - 4y + 4 = 0$ ..... equation of the circle

03. find equation of the circle having radius = 1 and touching the x – axis at (–4,0)

SOLUTION



CIRCLE 1 $C(-4,1)$, $r = 1$

$(x - h)^2 + (y - k)^2 = r^2$

$(x + 4)^2 + (y - 1)^2 = 1$

$x^2 + 8x + 16 + y^2 - 2y + 1 = 1$

$x^2 + y^2 + 8x - 2y + 16 = 0$

CIRCLE 2 $C(-4,-1)$, $r = 1$

$(x - h)^2 + (y - k)^2 = r^2$

$(x + 4)^2 + (y + 1)^2 = 1$

$x^2 + 8x + 16 + y^2 + 2y + 1 = 1$

$x^2 + y^2 + 8x + 2y + 16 = 0$ ans : $x^2 + y^2 + 8x \pm 2y + 16 = 0$

Q4.

01. Find circle touching both the axes and having radius 7

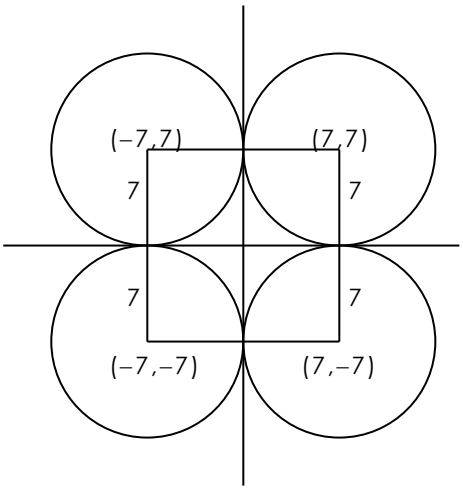
SOLUTION

Using : $(x - h)^2 + (y - k)^2 = r^2$

CIRCLE 1 $C(7,7)$, $r = 7$

$(x - 7)^2 + (y - 7)^2 = 7^2$

$x^2 + y^2 - 14x - 14y + 49 = 0$



CIRCLE 2 $C(-7,7)$, $r = 7$

$(x + 7)^2 + (y - 7)^2 = 7^2$

$x^2 + y^2 + 14x - 14y + 49 = 0$

CIRCLE 3 $C(-7,-7)$, $r = 7$

$(x + 7)^2 + (y + 7)^2 = 7^2$

$x^2 + y^2 + 14x + 14y + 49 = 0$

CIRCLE 4 $C(7,-7)$, $r = 7$

$(x - 7)^2 + (y + 7)^2 = 7^2$

$x^2 + y^2 - 14x + 14y + 49 = 0$ ans : $x^2 + y^2 \pm 14x \pm 14y + 49 = 0$

02. find equation of the circle touching both axes and passing through (1,2)

SOLUTION

STEP 1 : $CP = r$

$CP^2 = r^2$

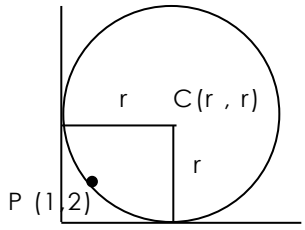
$(r-1)^2 + (r-2)^2 = r^2$

$r^2 - 2r + 1 + r^2 - 4r + 4 = r^2$

$r^2 - 6r + 5 = 0$

$(r-1)(r-5) = 0$

$r = 1 \;;\; r = 5$



STEP 2 :

$r = 1$; $C(1,1)$	$r = 5$; $C(5,5)$
$(x-h)^2 + (y-k)^2 = r^2$	$(x-h)^2 + (y-k)^2 = r^2$
$(x-1)^2 + (y-1)^2 = 1$	$(x-5)^2 + (y-5)^2 = 25$
$x^2 - 2x + 1 + y^2 - 2y + 1 = 1$	$x^2 - 10x + 25 + y^2 - 10y + 25 = 25$
$x^2 + y^2 - 2x - 2y + 1 = 0$	$x^2 + y^2 - 10x - 10y + 25 = 0$

03. find equation of the circle touching both axes and passing through (–9,8)

SOLUTION

STEP 1 : $CP = r$

$CP^2 = r^2$

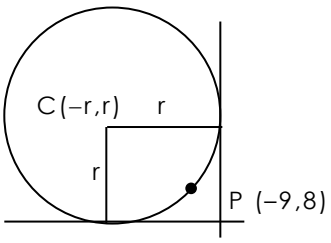
$(-r+9)^2 + (r-8)^2 = r^2$

$r^2 - 18r + 81 + r^2 - 16r + 64 = r^2$

$r^2 - 34r + 145 = 0$

$(r-5)(r-29) = 0$

$r = 5 \;;\; r = 29$



STEP 2 :

$r = 5$; $C(-5,5)$	$r = 29$; $C(-29,29)$
$(x-h)^2 + (y-k)^2 = r^2$	$(x-h)^2 + (y-k)^2 = r^2$
$(x+5)^2 + (y-5)^2 = 25$	$(x+29)^2 + (y-29)^2 = 841$
$x^2 + 10x + 25 + y^2 - 10y + 25 = 25$	$x^2 + 58x + 841 + y^2 - 58y + 841 = 841$
$x^2 + y^2 + 10x - 10y + 25 = 0$	$x^2 + y^2 + 58x - 58y + 841 = 0$

Q5.

01. Find equation of circle with center (4,3) & touching $5x - 12y - 10 = 0$

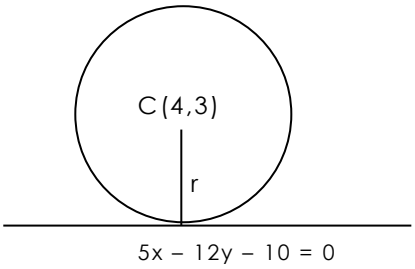
SOLUTION

STEP 1 : $r = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$

$= \left| \frac{5(4) - 12(3) - 10}{\sqrt{5^2 + 12^2}} \right|$

$= \left| \frac{20 - 36 - 10}{\sqrt{169}} \right|$

$= \left| \frac{-26}{13} \right| = 2$



STEP 2 : $C(4,3)$, $r = 2$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 4)^2 + (y - 3)^2 = (2)^2$

$x^2 - 8x + 16 + y^2 - 6y + 9 = 4$

$x^2 + y^2 - 8x - 6y + 25 - 4 = 0$

$x^2 + y^2 - 8x - 6y + 21 = 0$ ..... equation of the circle

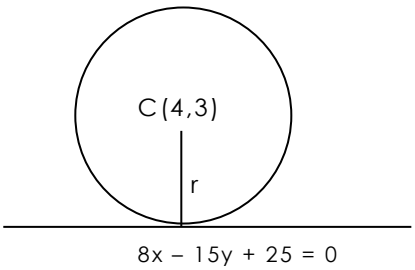
02. Find equation of circle with center (3,1) & touching $8x - 15y + 25 = 0$

STEP 1 : $r = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$

$= \left| \frac{8(3) - 15(1) + 25}{\sqrt{8^2 + 15^2}} \right|$

$= \left| \frac{24 - 15 + 25}{\sqrt{289}} \right|$

$= \left| \frac{34}{17} \right| = 2$



STEP 2 : $C(3,1)$, $r = 2$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 3)^2 + (y - 1)^2 = (2)^2$

$x^2 - 6x + 9 + y^2 - 2y + 1 = 4$

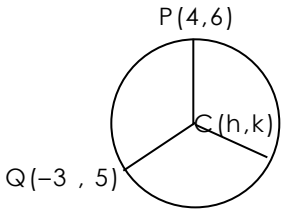
$x^2 + y^2 - 6x - 2y + 10 - 4 = 0$

$x^2 + y^2 - 6x - 2y + 6 = 0$ ..... equation of the circle

Q6.

01. Find equation of circle passing through (4 , 6) ; (–3 , 5) & (5 , –1)

SOLUTION



STEP 1 :

$CP = CQ$

$CP^2 = CQ^2$

$(h - 4)^2 + (k - 6)^2 = (h + 3)^2 + (k - 5)^2$

$h^2 - 8h + 16 + k^2 - 12k + 36 = h^2 + 6h + 9 + k^2 - 10k + 25$

$-8h - 12k + 52 = 6h - 10k + 34$

$18 = 14h + 2k$

$9 = 7h + k$ …… (1)

STEP 2 :

$CP = CR$

$CP^2 = CR^2$

$(h - 4)^2 + (k - 6)^2 = (h - 5)^2 + (k + 1)^2$

$h^2 - 8h + 16 + k^2 - 12k + 36 = h^2 - 10h + 25 + k^2 + 2k + 1$

$-8h - 12k + 52 = -10h + 2k + 26$

$2h - 14k = -26$

$h - 7k = -13$ …… (2)

STEP 3 : SOLVING (1) & (2)

$7 \times \quad 7h + k = 9$

$h - 7k = -13$

$49h + 7k = 63$

$h - 7k = -13$

$50h = 50$

$h = 1$

$k = 2$ $\qquad C \equiv (1 , 2)$

STEP 4 : C (1 , 2) , P(4,6)

$r = CP$

$= \sqrt{(1 - 4)^2 + (2 - 6)^2}$

$= \sqrt{25}$

$= 5$

STEP 5 : C(1 , 2) , r = 5

$(x - h)^2 + (y - k)^2 = r^2$

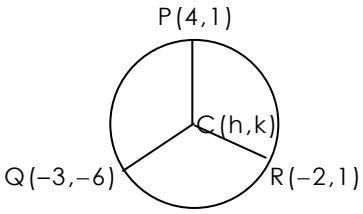
$(x - 1)^2 + (y - 2)^2 = 5^2$

$x^2 - 2x + 1 + y^2 - 4y + 4 = 25$

$x^2 + y^2 - 2x - 4y - 20 = 0$ …… Equation of circle

02. Find equation of circle passing through (4 , 1) ; (–3 , –6) & (–2 , 1)

SOLUTION



STEP 1 :

$CP = CQ$

$CP^2 = CQ^2$

$(h - 4)^2 + (k - 1)^2 = (h + 3)^2 + (k + 6)^2$

$h^2 - 8h + 16 + k^2 - 2k + 1 = h^2 + 6h + 9 + k^2 + 12k + 36$

$-8h - 2k + 17 = 6h + 12k + 45$

$-28 = 14h + 14k$

$-2 = h + k$ …… (1)

STEP 2 :

$CP = CR$

$CP^2 = CR^2$

$(h - 4)^2 + (k - 1)^2 = (h + 2)^2 + (k - 1)^2$

$h^2 - 8h + 16 + k^2 - 2k + 1 = h^2 + 4h + 4 + k^2 - 2k + 1$

$-8h - 2k + 17 = 4h - 2k + 5$

$12 = 12h$

$h = 1$ …… (2)

STEP 3 : Solving (1) & (2)

sub $h = 1$ in (1)

$k = -3$ $\quad C \equiv (1 , -3)$

STEP 4 : $C\ (1 , -3)$, $P(4,1)$

$r = CP$

$= \sqrt{(1 - 4)^2 + (-3 - 1)^2}$

$= \sqrt{9 + 16}$

$= 5$

STEP 5 : $C(1 , -3)$, $r = 5$

$(x - h)^2 + (y - k)^2 = r^2$

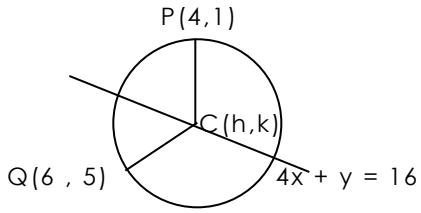
$(x - 1)^2 + (y + 3)^2 = 25$

$x^2 - 2x + 1 + y^2 + 6y + 9 = 25$

$x^2 + y^2 - 2x + 6y - 15 = 0$ …… Equation of circle

05. Find equation of circle passing through (4 , 1) ; (6 , 5) & whose center lies on $4x + y = 16$

SOLUTION



STEP 1 :

Since C(h , k) lies on $4x + y = 16$

$4h + k = 16$ …… (1)

STEP 2 :

$CP = CQ$

$CP^2 = CQ^2$

$(h - 4)^2 + (k - 1)^2 = (h - 6)^2 + (k - 5)^2$

$h^2 - 8h + 16 + k^2 - 2k + 1 = h^2 - 12h + 36 + k^2 - 10k + 25$

$-8h - 2k + 17 = -12h - 10k + 61$

$4h + 8k = 44$

$h + 2k = 11$ …… (2)

STEP 3 : $2 \times \quad 4h + k = 16$

$h + 2k = 11$

$8h + 2k = 32$

$h + 2k = 11$

$7h = 21$

$h = 3$

$k = 4$ $\qquad C \equiv (3 , 4)$

STEP 4 : C (3 , 4) , P(4,1)

$r = CP$

$= \sqrt{(3 - 4)^2 + (4 - 1)^2}$

$= \sqrt{1 + 9}$

$= \sqrt{10}$

STEP 5 : C(3 , 4) , $r = \sqrt{10}$

$(x - h)^2 + (y - k)^2 = r^2$

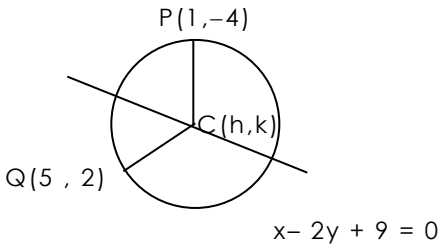
$(x - 3)^2 + (y - 4)^2 = 10$

$x^2 - 6x + 9 + y^2 - 8y + 16 = 10$

$x^2 + y^2 - 6x - 8y + 15 = 0$ ……. Equation of circle

06. Find equation of circle passing through (1 , –4) ; (5 , 2) & whose center lies on x – 2y + 9 = 0

SOLUTION



STEP 1 :

Since C(h , k) lies on x – 2y + 9 = 0

$h - 2k = -9$ …… (1)

STEP 2 :

CP = CQ

$CP^2 = CQ^2$

$(h-1)^2 + (k+4)^2 = (h-5)^2 + (k-2)^2$

$h^2 - 2h + 1 + k^2 + 8k + 16 = h^2 - 10h + 25 + k^2 - 4k + 4$

$-2h + 8k + 17 = -10h - 4k + 29$

$8h + 12k = 12$

$2h + 3k = 3$ …… (2)

STEP 3 : $2 \times \quad h - 2k = -9$ $\qquad 2h - 4k = -18$

$2h + 3k = 3$ $\qquad 2h + 3k = 3$

$-7k = -21$

$k = 3$

subs in (1) $h = -3$ $\qquad C \equiv (-3 , 3)$

STEP 4 : C (–3 , 3) , P(1,–4)

$r = CP$

$= \sqrt{(-3-1)^2 + (3+4)^2}$

$= \sqrt{16 + 49}$

$= \sqrt{65}$

STEP 5 : C(–3 , 3), $r = \sqrt{65}$

$(x-h)^2 + (y-k)^2 = r^2$

$(x+3)^2 + (y-3)^2 = 65$

$x^2 + 6x + 9 + y^2 - 6y + 9 = 65$

$x^2 + y^2 + 6x - 6y + 18 - 65 = 0$

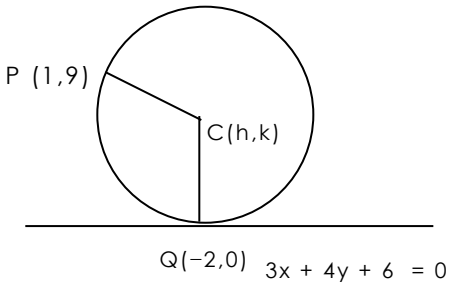
$x^2 + y^2 + 6x - 6y - 47 = 0$ …… Equation of circle

Q7.

01. find equation of circle passing through (1 , 9) & touching $3x + 4y + 6 = 0$ at (–2 , 0)

SOLUTION

STEP 1 :



$CP = CQ$

$CP^2 = CQ^2$

$(h - 1)^2 + (k - 9)^2 = (h + 2)^2 + (k - 0)^2$

$h^2 - 2h + 1 + k^2 - 18k + 81 = h^2 + 4h + 4 + k^2$

$-2h - 18k + 82 = 4h + 4$

$78 = 6h + 18k$

$h + 3k = 13$ …… (1)

STEP 2 :

Slope of line

$3x + 4y + 6 = 0$: $m = -\frac{a}{b} = -\frac{3}{4}$

$\therefore m_{CQ} = \frac{4}{3}$ ……. (Tangent – Radius)

Now ; $m_{CQ} = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{4}{3} = \frac{k - 0}{h + 2}$

$4h + 8 = 3k$

$4h - 3k = -8$ ……. (2)

STEP 3 :

Solving (1) & (2)

$h + 3k = 13$

$4h - 3k = -8$

$5h = 5$

$h = 1$

sub in (1) , $k = 4$

C(1,4)

STEP 4 : $r = CQ$, C(1,4) , Q(–2,0)

$= \sqrt{(1 + 2)^2 + (4 - 0)^2}$

$= 5$

STEP 5 : C(1 , 4) , r = 5

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 1)^2 + (y - 4)^2 = 25$

$x^2 - 2x + 1 + y^2 - 8y + 16 = 25$

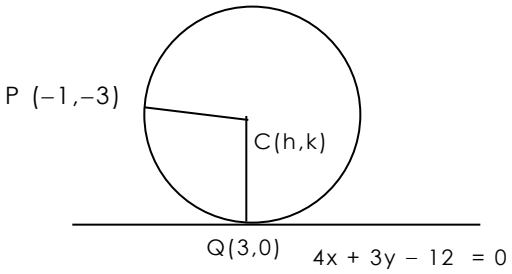
$x^2 + y^2 - 2x - 8y + 17 - 25 = 0$

$x^2 + y^2 - 2x - 8y - 8 = 0$ ……. Equation of circle

02. find equation of circle passing through (–1 , –3) & touching $4x + 3y – 12 = 0$ at (3 , 0)

SOLUTION

STEP 1 :



$CP = CQ$

$CP^2 = CQ^2$

$(h + 1)^2 + (k + 3)^2 = (h – 3)^2 + (k – 0)^2$

$h^2 + 2h + 1 + k^2 + 6k + 9 = h^2 – 6h + 9 + k^2$

$2h + 6k + 10 = – 6h + 9$

$8h + 6k = – 1$ ..... (1)

STEP 2 :

Slope of line

$4x + 3y – 12 = 0$: $m = -\frac{a}{b} = -\frac{4}{3}$

$\therefore m_{CQ} = \frac{3}{4}$ ..... (Tangent – Radius)

Now ; $m_{CQ} = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{3}{4} = \frac{k - 0}{h - 3}$

$3h – 9 = 4k$

$3h – 4k = 9$ ..... (2)

STEP 3 :

Solving (1) & (2)

(1) x 2 $16h + 12k = –2$

(2) x 3 $9h – 12k = 27$

$25h = 25$

$h = 1$

sub in (1) , $k = \frac{-3}{2}$

C (1 , –3/2)

STEP 4 : $r = CQ$, C(1,–3/2) , Q(3,0)

$= \sqrt{(1 – 3)^2 + (–3/2 – 0)^2}$

$= \sqrt{4 + \frac{9}{4}} = \sqrt{\frac{25}{4}} = \frac{5}{2}$

STEP 5 : C(1 , –3/2) , r = 5/2

$(x – h)^2 + (y – k)^2 = r^2$

$(x – 1)^2 + (y + 3/2)^2 = 25/4$

$x^2 – 2x + 1 + y^2 + 3y + \frac{9}{4} = \frac{25}{4}$

$\frac{4x^2 – 8x + 4 + 4y^2 + 12y + 9}{4} = \frac{25}{4}$

$4x^2 + 4y^2 – 8x + 12y + 13 – 25 = 0$

$4x^2 + 4y^2 – 8x + 12y – 12 = 0$

$x^2 + y^2 – 2x + 3y – 3 = 0$ ..... Equation of circle

Q8.

01. Find equation of circle with center (3 , –1) and which cuts off a chord of length 6 on line $2x - 5y + 18 = 0$

SOLUTION

STEP 1 : AP = PB = 3 ($\perp$ from the centre bisects the chord)

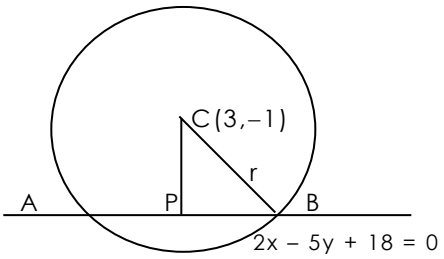
STEP 2 : $CP = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$

$= \left| \frac{2(3) - 5(-1) + 18}{\sqrt{2^2 + 5^2}} \right|$

$= \left| \frac{6 + 5 + 18}{\sqrt{29}} \right|$

$= \left| \frac{29}{\sqrt{29}} \right|$

$CP = \sqrt{29}$



STEP 3 : In Δ CPB ; $CP^2 + PB^2 = r^2$

$29 + 9 = r^2$

$r^2 = 38$

$r = \sqrt{38}$

STEP 4 : C(3 , –1), $r = \sqrt{38}$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 3)^2 + (y + 1)^2 = 38$

$x^2 - 6x + 9 + y^2 + 2y + 1 = 38$

$x^2 + y^2 - 6x + 2y + 10 - 38 = 0$

$x^2 + y^2 - 6x + 2y - 28 = 0$ ..... Equation of circle

02. Find equation of circle with center (1 , 4) and which cuts off a chord of length 6 on line $3x + 4y + 1 = 0$

SOLUTION

STEP 1 : AP = PB = 3 ($\perp$ from the centre bisects the chord)

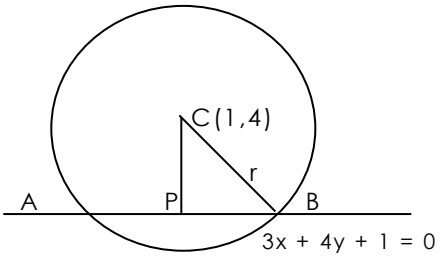
STEP 2 : $CP = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$

$= \left| \frac{3(1) + 4(4) + 1}{\sqrt{3^2 + 4^2}} \right|$

$= \left| \frac{3 + 16 + 1}{\sqrt{25}} \right|$

$= \left| \frac{20}{5} \right|$

$CP = 4$



STEP 3 : In Δ CPB ; $CP^2 + PB^2 = r^2$

$16 + 9 = r^2$

$r^2 = 25$

$r = 5$

STEP 4 : C(1 , 4), r = 5

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 1)^2 + (y - 4)^2 = 25$

$x^2 - 2x + 1 + y^2 - 8y + 16 = 25$

$x^2 + y^2 - 2x - 8y + 17 - 25 = 0$

$x^2 + y^2 - 2x - 8y - 8 = 0$ ..... Equation of circle

03. Find the length of intercept made by circle $x^2 + y^2 - 2x - 8y - 8 = 0$ on the line $3x + 4y + 1 = 0$

SOLUTION

STEP 1 : $x^2 + y^2 - 2x - 8y - 8 = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -2 \; ; \quad 2f = -8$

$g = -1 \; ; \quad f = -4 \; ; \quad c = -8$

$C \equiv (-g \, , -f) \qquad r = \sqrt{g^2 + f^2 - c}$

$\equiv (1 \, , 4) \qquad = \sqrt{1 + 16 + 8}$

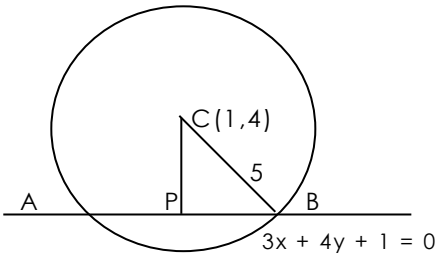
$= 5$

STEP 2 : $CP = \left| \dfrac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$

$= \left| \dfrac{3(1) + 4(4) + 1}{\sqrt{3^2 + 4^2}} \right|$

$= \left| \dfrac{3 + 16 + 1}{\sqrt{25}} \right|$

$= \left| \dfrac{20}{5} \right|$



$CP = 4$

STEP 3 : In Δ CPB ; $CP^2 + PB^2 = r^2$

$16 + PB^2 = 25$

$PB^2 = 9$

$PB = 3$

STEP 4 : $AB = 2(PB) = 6$ ..... ($\perp$ from the centre bisects the chord)

04. Find the length of intercept made by circle $x^2 + y^2 - 6x + 4y - 12 = 0$ on the line $4x - 3y + 2 = 0$

SOLUTION

STEP 1 : $x^2 + y^2 - 6x + 4y - 12 = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -6 \; ; \quad 2f = 4$

$g = -3 \; ; \quad f = 2 \; ; \quad c = -12$

$C \equiv (-g \, , -f)$ $\qquad r = \sqrt{g^2 + f^2 - c}$

$\equiv (3 \, , -2)$ $\qquad = \sqrt{9 + 4 + 12}$

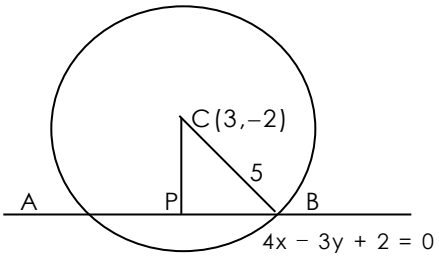
$= 5$

STEP 2 : $CP = \left| \dfrac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$

$= \left| \dfrac{4(3) - 3(-2) + 2}{\sqrt{3^2 + 4^2}} \right|$

$= \left| \dfrac{12 + 6 + 2}{\sqrt{25}} \right|$

$= \left| \dfrac{20}{5} \right|$



$CP = 4$

STEP 3 : In Δ CPB ; $CP^2 + PB^2 = r^2$

$16 + PB^2 = 25$

$PB^2 = 9$

$PB = 3$

STEP 4 : $AB = 2(PB) = 6$ ..... ($\perp$ from the centre bisects the chord)

MARCH – 2017

Find equation of circle passing through point of intersection of the lines $x + 3y = 0$ and $2x - 7y = 0$ and whose centre is the point of intersection of the lines $x + y + 1 = 0$ and $x - 2y + 4 = 0$

SOLUTION

STEP 1

point of intersection of the lines $x + 3y = 0$ and $2x - 7y = 0 \equiv (0,0)$

STEP 2

point of intersection of the lines $x + y + 1 = 0$ and $x - 2y + 4 = 0 \equiv (-2,1)$

STEP 3

Centre of the circle is $(-2,1)$ and circle passes through $(0,0)$

Radius $r = \sqrt{(-2-0)^2 + (1-0)^2}$

$= \sqrt{4+1}$

$= \sqrt{5}$

STEP 4 Equation of the circle $(x-h)^2 + (y-k)^2 = r^2$

$(x+2)^2 + (y-1)^2 = 5$

$x^2 + y^2 + 4x - 2y = 0$